

Exercise #1: Horizontal foreign direct investment

1. Optimal price of exporter derives from profit maximization. The profit of the exporter writes:

$$\pi_i = p_i y_i - w_i \beta y_i = (p_i - w_i \beta) y_i = (p_i - w_i \beta) c_{ij} T_{ij} = (p_i - w_i \beta) \left(\frac{p_i T_{ij}}{P_j} \right)^{-\sigma} \frac{Y_j}{P_j} T_{ij}$$

To calculate the FOC, it is convenient to re-write the profit as:

$$\pi_i = p_i^{1-\sigma} \left(\frac{T_{ij}}{P_j} \right)^{-\sigma} \frac{Y_j}{P_j} T_{ij} - w_i \beta p_i^{-\sigma} \left(\frac{T_{ij}}{P_j} \right)^{-\sigma} \frac{Y_j}{P_j} T_{ij}$$

FOC:

$$\frac{\partial \pi_i}{\partial p_i} = (1-\sigma) p_i^{-\sigma} \left(\frac{T_{ij}}{P_j} \right)^{-\sigma} \frac{Y_j}{P_j} T_{ij} + \sigma w_i \beta p_i^{-\sigma-1} \left(\frac{T_{ij}}{P_j} \right)^{-\sigma} \frac{Y_j}{P_j} T_{ij} = 0$$

The optimal price p_i then is:

$$\boxed{p_i = \frac{\sigma}{\sigma-1} w_i \beta}$$

Substituting w_i by its function of p_i in the expression of profit yields:

$$\pi_i = \left(1 - \frac{\sigma-1}{\sigma} \right) p_i y_i = \frac{p_i y_i}{\sigma}$$

$$\boxed{\pi_i = \frac{1}{\sigma} \left(\frac{p_i T_{ij}}{P_j} \right)^{1-\sigma} Y_j}$$

Comments:

- The optimal price is a markup ($\sigma/\sigma-1$) on the marginal cost ($w_i \beta$); when the elasticity of substitution goes to infinity, then the markup is one (perfect competition).
- Higher transportation costs reduce the profit from exporting, whereas higher GDP abroad increases profit. When the elasticity of substitution goes to infinity, profit goes to zero (perfect competition).

2. Optimal price when locating abroad again is given by profit maximization. The profit now writes:

$$\pi_i = p_i y_i - w_i \alpha - w_i \beta y_i = (p_i - w_i \beta) y_i - w_i \alpha = (p_i - w_i \beta) \left(\frac{p_i}{P_j} \right)^{-\sigma} \frac{Y_j}{P_j} T_{ij} - w_i \alpha$$

The optimal price is the same as for the exporter: $p_i = \frac{\sigma}{\sigma-1} w_i \beta$

The profit can then be written: $\pi_i = \left(1 - \frac{\sigma-1}{\sigma}\right) p_i y_i - \frac{\sigma-1}{\sigma\beta} p_i \alpha$

$$\pi_i = \frac{1}{\sigma} \left(\frac{p_i}{P_j}\right)^{1-\sigma} Y_j - \frac{\sigma-1}{\sigma\beta} p_i \alpha$$

Comments:

- The optimal price is the same as for the exporter because in this model, the exporter is able to pass the transportation cost on the importer.
- The profit no longer depends on the transportation cost. It is positively related to foreign income and negatively to the fixed cost. Note that, when $\sigma \rightarrow \infty$ (perfect competition), profit is negative due to the fixed cost: only with product differentiation can a foreign firm cover the fixed cost of producing in the target country.

3. Condition for an exporter from i to switch to local production in j

$$\frac{1}{\sigma} \left(\frac{p_i T_{ij}}{P_j}\right)^{1-\sigma} Y_j < \frac{1}{\sigma} \left(\frac{p_i}{P_j}\right)^{1-\sigma} Y_j - \frac{\sigma-1}{\sigma\beta} p_i \alpha$$

Re-arranging, we get the following condition:

$$T_{ij}^{1-\sigma} < 1 - \frac{\sigma-1}{\beta} \frac{p_i \alpha}{Y_j} \left(\frac{p_i}{P_j}\right)^{\sigma-1}$$

The condition is met more easily if:

- The transportation cost is high (because exporting inflates the cost for the foreign consumer, hence reducing his demand)
- The fixed cost α is low (because α reduces the profit of producing abroad)
- Foreign income is high (because a rise in Y_j has more impact on the profit made by a foreign subsidiary than by the profit made the foreign exporter: the share of foreign income which is allocated to good i is higher in the former case because the price of i is lower due to the absence of transportation costs)

Symmetrically, the condition for an exporter from j to switch to local production in i is:

$$T_{ji}^{1-\sigma} < 1 - \frac{\sigma-1}{\beta} \frac{p_j \alpha}{Y_i} \left(\frac{p_j}{P_i}\right)^{\sigma-1}$$

4. For a multinational to operate in both countries, the two inequalities must hold simultaneously. Multiplying them together, we get the following *necessary* condition:

$$(T_{ij}T_{ji})^{1-\sigma} < 1 - (\sigma-1) \frac{p_i \alpha}{Y_j} \left(\frac{p_i}{P_j}\right)^{\sigma-1} - \frac{\sigma-1}{\beta} \frac{p_j \alpha}{Y_i} \left(\frac{p_j}{P_i}\right)^{\sigma-1} - \left(\frac{\sigma-1}{\beta}\right)^2 \left(\frac{p_i p_j \alpha^2}{Y_i Y_j}\right) \left(\frac{p_i p_j}{P_j P_i}\right)^{\sigma-1}$$

Horizontal multinationals are more likely when (i) transportation costs are higher, (ii) plant-specific fixed costs are lower, and (iii) GDPs are higher in both countries.

Substituting p_i by its function of w_i yields:

$$(T_{ij}T_{ji})^{1-\sigma} < 1 - \frac{\sigma w_i \alpha}{Y_j} \left(\frac{\sigma \beta w_i}{(\sigma-1)P_j}\right)^{\sigma-1} - \frac{\sigma w_j \alpha}{Y_i} \left(\frac{\sigma \beta w_j}{(\sigma-1)P_i}\right)^{\sigma-1} - \frac{\sigma^2 w_i w_j \alpha^2}{Y_i Y_j} \left(\frac{\sigma^2 \beta^2 w_i w_j}{P_j P_i}\right)^{\sigma-1}$$

Labor costs have a negative impact on the probability that this condition is satisfied.

5. Horizontal multinational more likely in more similar countries in terms of income

With $Y = Y_i + Y_j$, $s_i = Y_i/Y$ and $s_j = Y_j/Y$, we have:

$$Y_i Y_j = s_i s_j Y^2$$

s_j varies from 0 (one of the two countries has very small income) to 0.25 ($s_i = s_j = 0.5$). For a given Y , the closer the two incomes, the larger the product $Y_i Y_j$. Since a higher $Y_i Y_j$ raises the right-hand side of the inequality, it can be concluded that a horizontal multinational is more likely to operate in similar countries in terms of income, other things being equal.

Properties 4 and 5 were exhibited by Markusen and Venables (1998, 2000) and Markusen (2002).

6. The table shows the impact of each variable on the probability of a French investor to select one country. We recognize the impact of size (market access) and transportation costs (distance). GDP per capita represents real wages. Supply access corresponds to upstream and downstream externalities. Agglomeration effects are captured by the number of French firms of the same industry already located in the country.

Market access: $MA_i = \sum_j \phi_{ij} E_j P_i^{\sigma-1}$, where E_j is country j 's total expenditure, P_j is its price index

(competition faced in country j) and ϕ_{ij} is ease of access to market j from country i ($0 < \phi_{ij} < 1$).

Other things being equal, the probability of investing in France is higher. But this is less the case for larger or more productive firms.

Exercise #2: Vertical foreign direct investment

1. Profit when producing in both countries

$$\Pi = 10 \times (40 + 5) - 7 \times 40 - 6 \times 5 - 2 \times 30 = 450 - 280 - 30 - 60 = 80$$

2. Profit when producing only in France

$$\Pi = 10 \times (40 + 5) - 7 \times 40 - 7 \times 5 - 30 - 5t = 450 - 280 - 35 - 30 - 5t = 105 - 5t$$

Profit when producing only in Germany

$$\Pi = 10 \times (40 + 5) - 6 \times 40 - 6 \times 5 - 30 - 40t = 450 - 240 - 30 - 30 - 40t = 150 - 40t$$

3. Trade costs of 6 M€ per plane

Profit when producing only in France: $\Pi = 105 - 5 \times 6 = 105 - 30 = 75$

Profit when producing only in Germany: $\Pi = 150 - 40 \times 6 = 150 - 240 = -90$

Profit when producing in both countries: $\Pi = 80$

Producing only in Germany yields a negative profit because transportation costs are too high. Producing only in France is not worth incurring the fixed cost of a second plant.

4. Trade costs of 1 M€ per plane

Profit when producing only in France: $\Pi = 105 - 5 \times 1 = 105 - 5 = 95$

Profit when producing only in Germany: $\Pi = 150 - 40 \times 1 = 150 - 40 = 110$

Profit when producing in both countries: $\Pi = 80$

Producing only in Germany yields more profit because now transportation costs are so low that it is worth relocating in Germany.

Note that, with trade costs of 2 M€ per plane, producing only in France yields more profit: the firm locates where the market is bigger, despite higher transportation costs:

Profit when producing only in France: $\Pi = 105 - 5 \times 2 = 105 - 10 = 95$

Profit when producing only in Germany: $\Pi = 150 - 40 \times 2 = 150 - 80 = 70$

Profit when producing in both countries: $\Pi = 80$