PC3 corrected

Exercise #1: gains of trade under imperfect competition

1. Increasing returns (2) are not consistent with perfect competition since the average cost would then be higher than the marginal cost and firms would each register a loss $Y_i(p - \frac{f + cY_i}{Y_i}) = -f$.

   We thus have to make the assumption (1) of monopolistic competition. Given the implied demand curve, if farmers set identical prices, they will have identical market shares equal to *1/N*. Coefficient *b* measures demand sensitivity to deviations of prices from their average. The model does not specify the consumption function but it could be derived from a utility function with preference for diversity, or with search for an ideal variety. Average cost (AC_i) increases with the number of farms, since smaller farms bear an unchanged fixed cost:

$$\boxed{AC_i = \frac{C_i}{Y_i} = c + \frac{f}{Y_i} = c + N\frac{f}{\overline{\overline{Y}}}}.$$

   This is curve (CC) on the figure.

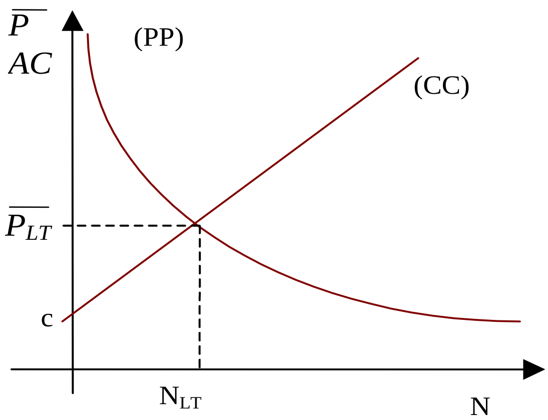
2. Perceived demand curve: $P_i - \overline{P} = \frac{1}{bN} - \frac{Y_i}{b\overline{\overline{Y}}}$

   Nominal revenue: $R_i = P_i\, Y_i$

   Marginal revenue: $\frac{\partial R_i}{\partial Y_i} = P_i + Y_i \frac{\partial P_i}{\partial Y_i} = P_i - \frac{Y_i}{b\overline{\overline{Y}}}$

   Marginal revenue = marginal cost: $c = P_i + Y_i \frac{\partial P_i}{\partial Y_i} = P_i - \frac{Y_i}{b\overline{\overline{Y}}}$

   In a symmetrical equilibrium, we have: $\frac{Y_i}{\overline{\overline{Y}}} = \frac{1}{N}$, thus: $\boxed{\overline{P} = c + \frac{1}{bN}}$. Price is higher than marginal cost *c*; markup $\overline{P} - c$ decreases when the number of competitors is higher (curve (PP) on the figure).
   The intercept between (CC) and (PP) gives the long-run equilibrium (free entry). In the short run, the equilibrium may not be on the (CC) curve, in which case profit is different from zero, triggering entry or exit of firms.



3. In the long run, the number of farms adjusts so that profit is equal to zero, *i.e.* price is equal to average cost (curve (CC) on the figure). The long-term equilibrium is thus at the intersection of (PP) and (CC). We then have $c + N\frac{f}{\overline{Y}} = c + \frac{1}{bN}$ thus $\boxed{N = \sqrt{\frac{\overline{Y}}{bf}}}$. The number of farms increases less than one-to-one with the overall size of the market. Equilibrium price $\boxed{\overline{P} = c + \frac{1}{bN} = c + \sqrt{\frac{f}{b\overline{Y}}}}$ decreases with market size. Individual output $\boxed{Y_i = \frac{\overline{Y}}{N} = \sqrt{bf\overline{Y}}}$ increases with overall market size and with fixed cost: for a given market size, a higher cost means that production needs to be more concentrated to cover the fixed cost.

4. After trade has been allowed, curve (PP) does not move since the price equation does not depend on market size. Average-cost curve (CC) moves downwards: for a given number of farms *N*, average cost is lower because farms can now sell apples to twice as many customers as before. But this incites new farmers to join.

   1→2 (market openness): in the short run, the number of farms stays constant in each country, so $\mathcal{N} = 2\sqrt{\frac{\overline{Y}}{bf}}$. For the initial market price, however, there is too much supply compared to demand.

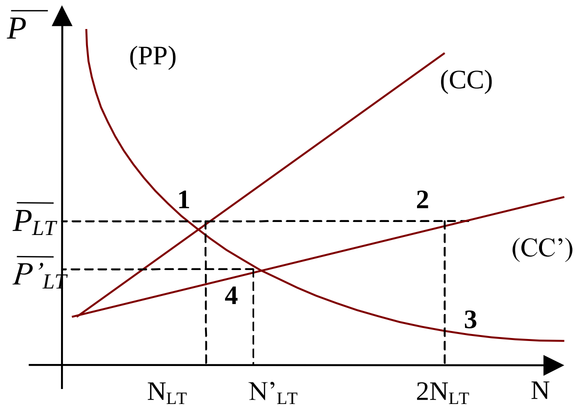
   2→3 (price adjustment): hence the unit price falls until a short-run, market equilibrium is found in Point 3. At this point, however, the unit price is lower than the average cost: profit is negative.

   At this point, if both governments step in and subsidize the difference between price and average cost so that farmers are break-even, the economy will never move to its long-term equilibrium 4, which is welfare improving.

   3→4 (entry and exit): absent government intervention, the number of farms decreases, which pushes the price higher and diminishes the average cost until farms do not lose money anymore. This is the long-term equilibrium where price = marginal revenue = average cost.

   - The number of firms is now $\boxed{\mathcal{N} = \sqrt{\frac{2\overline{Y}}{bf}}}$
   - Each firm produces $\boxed{Y'_i = \overline{Y}/\mathcal{N} = \sqrt{\frac{\overline{Y}}{2bf}} = Y_i\sqrt{2} > Y_i.}$ (scale economies)
   - The unit price is now such as $\overline{P} = c + 1/b\mathcal{N}$, so $\boxed{\overline{P}' = c + \sqrt{\frac{f}{2b\overline{Y}}} < \overline{P}}$ (lower markup).

   Note that contrasting with Krugman (1980), the markup rate depends on the number of varieties. This is why the number of firms is eventually lower than 2N.



Exercise #2: the gravity model

1. Budget contraint: $Y_j = \sum_{i=1}^{C} N_i p_{ij} c_{ij}$

Lagrangian:

$$L_j = \left[\sum_{i=1}^{C} N_i \left(c_{ij}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - \lambda \left[\sum_{i=1}^{C} N_i - p_{ij} c_{ij} Y_j \right]$$

First derivative with respect to c_{ij} :

$$c_{ij}^{-1/\sigma} = \lambda p_{ij} U_j^{-1/\sigma} \qquad \text{hence} \qquad c_{ij} = \lambda^{-\sigma} p_{ij}{}^{-\sigma} U_j$$

which can be rewritten:

$$N_i p_{ij} c_{ij} = N_i \lambda^{-\sigma} p_{ij}{}^{1-\sigma} U_j$$

which can be summed over *i*:

$$\sum_{i=1}^{C} N_i p_{ij} c_{ij} = \lambda^{-\sigma} U_j \sum_{i=1}^{C} N_i p_{ij}{}^{1-\sigma}$$

Y_j is on the LHS and $P_j^{1-\sigma}$ is on the RHS: $Y_j = \lambda^{-\sigma} U_j P_j^{1-\sigma}$.

λ can now be replaced by its value as given by $c_{ij}^{-1/\sigma} = \lambda p_{ij} U_j^{-1/\sigma}$:

$$Y_j = c_{ij} p_{ij}^{\sigma} P_j^{1-\sigma}$$

hence: $$\boxed{c_{ij} = \left(\frac{p_{ij}}{P_j} \right)^{-\sigma} \frac{Y_j}{P_j}}$$

2. $$c_{ij} = \left(\frac{(1+\tau_{ij}) p_{ij}}{P_j} \right)^{-\sigma} \frac{Y_j}{P_j}$$

Exports of good *i* to country *j* thus depend on revenue in country *j*, relative price of good *i*, and transport cost of *i* to *j*. Other things equal, a 1% increase in the transportation cost reduces consumption of the good by σ%. However, P_j also depends on τ_{ij}, so the *ceteris paribus* condition does not hold. Econometricians prefer to work on the c_{ij}/c_{jj} ratio.

3. From what precedes, the CIF value exports of good *i* to country *j* writes:

$$X_{ij} = N_i(1+\tau_{ij})p_{ij}c_{ij} = \left(\frac{(1+\tau_{ij})p_{ij}}{P_j}\right)^{1-\sigma} N_i Y_j$$

thus in logarithm:

$$\ln X_{ij} = (1-\sigma)[(\ln(1+\tau_{ij}) + \ln p_{ij} - \ln P_j] + \ln(N_i) + \ln(Y_j)$$

yielding the following equation to be estimated:

$$\boxed{\ln X_{ij} = a_0 + a_1 \ln N_i + a_2(1+\tau_{ij}) + a_3 \ln p_{ij} + a_4 \ln P_j + a_5 \ln Y_j + u_{ij}}$$

Theory predicts that $a_1=1$, $a_2<0$, $a_3<0$, $a_4>0$ and $a_5=1$. $(1+\tau_{ij})$ represents transport costs. It can be proxied by a set of qualitative or quantitative variables: distance, common border, common language, colonial past, etc. P_j represents "internal resistance" to trade. It is proxied by trade barriers, the belonging to a free trade agreement or a fixed j×t effect. As for p_{ij}, it is generally assumed that $p_{ij} = p_i$. The effect of p_i is then captured by GDP per capita or a fixed i×t effect.

4. One adds the relevant explanatory variable:

$$\ln X_{ij} = a_0 + a_1 \ln N_i + a_2 dist_{ij} + a_3 \ln p_i + a_4 \ln P_j + a_5 \ln Y_j + a_6 F_{ij} + u_{ij}$$

where F_{ij} is a dummy variable equal to one if countries *i* and *j* share a common currency. The impact on bilateral trade can be found in the equation expressed in level:

$$X_{ij} = e^{a_0} N_i^{a_1} dist_i^{a_2} p_i^{a_3} P_j^{a_4} Y_j^{a_5} e^{a_6 F_{ij}}$$

which yields the conditional impact of a having common currency:

$$\frac{X_{ij}(F_{ij}=1)}{X_{ij}(F_{ij}=0)} = e^{a_6}$$

Here $a_6 = 0.5$. All other things equal, merchandise trade is $e^{0.5} = 1.6$ times higher between countries which share a common currency.

Impact of monetary unions: Andy Rose (2000) has found $a_6 = 1.21$, hence a MU effect of 3.35. But there are many empirical issues: reverse causality, bias created by small MUs (e.g. in the Caribbean). More recent estimations: $a_6=0.7$ implying a MU effect of 2.