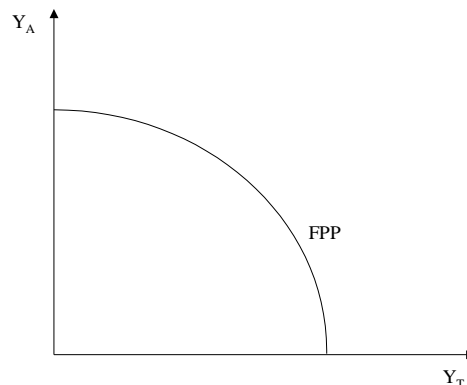


Exercise #1: Gains of trade

1) Autarky

a. The PPF is the locus of all outcomes (Y_T, Y_A) such that $L_A + L_T = L$. Note that:

$$Y_A = L_A^{\alpha_A} = (L - L_T)^{\alpha_A} = (L - Y_T^{1/\alpha_T})^{\alpha_A}, \text{ which implies: } \frac{\partial Y_A}{\partial Y_T} = -\frac{\alpha_A}{\alpha_T} \frac{L_A^{\alpha_A-1}}{L_T^{\alpha_T-1}} < 0 \text{ et } \frac{\partial^2 Y_A}{\partial Y_T^2} < 0$$



b. In a perfectly competitive economy, the FOC of profit maximization implies that marginal cost equals marginal productivity:

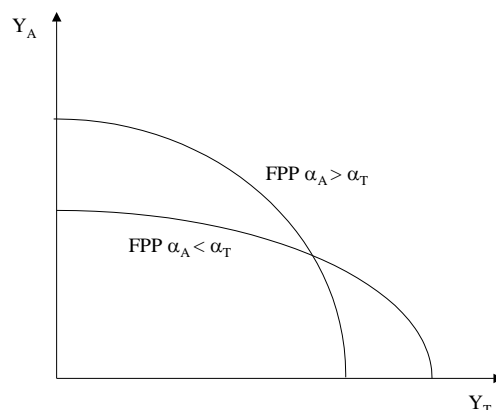
$$W/P_A = \partial Y_A / \partial L_A = \alpha_A L_A^{\alpha_A-1} \text{ et } W/P_T = \partial Y_T / \partial L_T = \alpha_T L_T^{\alpha_T-1}$$

Since nominal wages are equal, the ratio of real wages yields the relative price p :

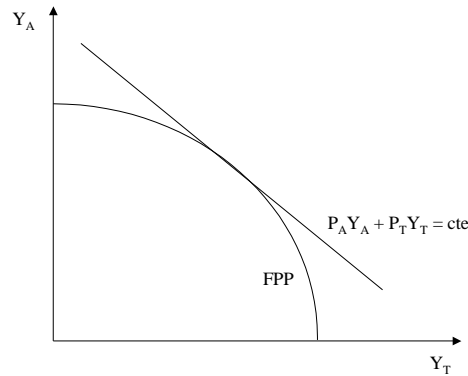
$$p = \frac{P_T}{P_A} = \frac{\alpha_A}{\alpha_T} \frac{L_A^{\alpha_A-1}}{L_T^{\alpha_T-1}} = -\frac{\partial Y_A}{\partial Y_T}$$

The marginal transformation rate (MTR) between the two goods is thus equal to their relative price. MTR is key to the comparative-advantage approach to international trade. When it comes to opening up the economy, the key issue is how much good A the economy would produce out of labor currently employed to produce good T.

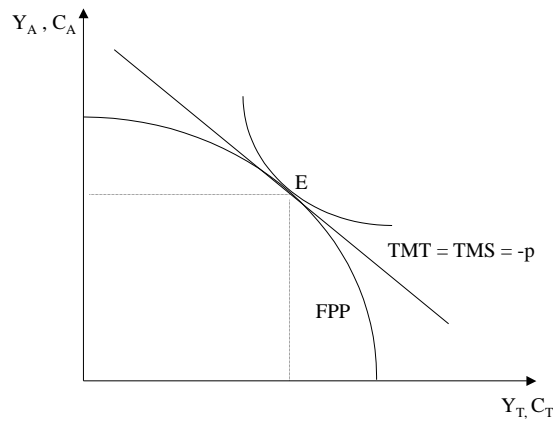
If $0 < \alpha_A < \alpha_T < 1$, textile is more labor-intensive than aircraft and the PPF is tilted to the right.



c. The isovalue line is defined by: $P_A Y_A + P_T Y_T = \text{constant}$. It is thus tangent to the PPF.



d. The consumer's FOC equals the marginal substitution rate (MSR) and relative price, and therefore MSR and MTR:
$$\text{MSR} = \frac{\partial U / \partial C_T}{\partial U / \partial C_A} = \frac{1-\beta}{\beta} \frac{C_A}{C_T} = \frac{P_T}{P_A} = p$$



Moreover, each good's consumption equals its production since the economy is closed.

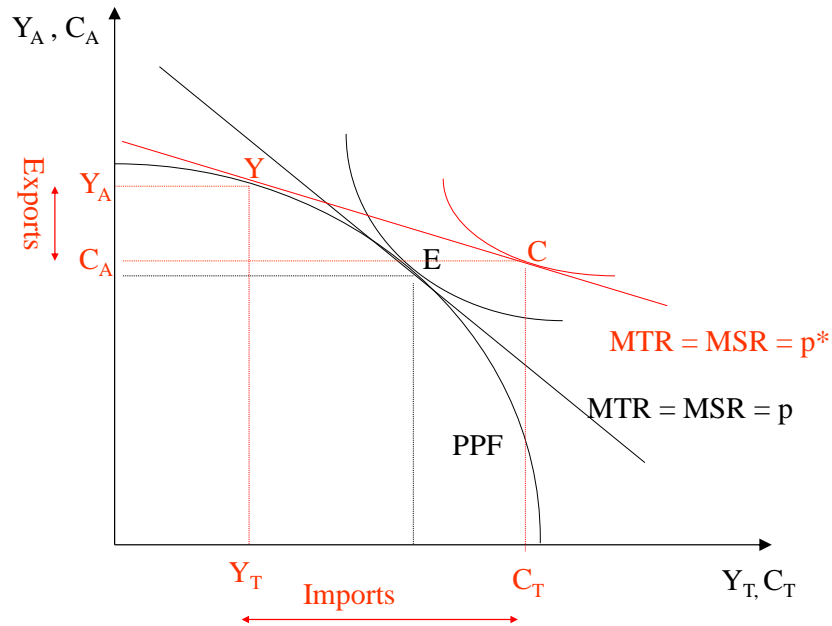
2) The open economy

a) Production is optimal if $\text{MTR} = p^*$:
$$p^* = \frac{\alpha_A L_A^{\alpha_A - 1}}{\alpha_T L_T^{\alpha_T - 1}}$$

Consumption is optimal if $\text{MSR} = p^*$:
$$p^* = \frac{1-\beta}{\beta} \frac{C_A}{C_T}$$

Trade is balanced if:
$$p_T^*(Y_T - C_T) + p_A^*(Y_A - C_A) = 0$$

b) Equilibrium with trade flows is represented below in the case where $p^* < \hat{p}$. The country specializes into aircraft: its exports airplanes and imports clothes. Specialization makes it possible for producers to produce less textile and more airplanes, and for consumers to consume more of each product.



Exercise #2: Comparative advantages

Question "1

- **North**

PPF : $4000 = 2 y_{1N} + 4 y_{2N}$, or else $y_{1N} = 2000 - 2 y_{2N}$.

$p_N = p_{2N}/p_{1N} = TMT = 2$.

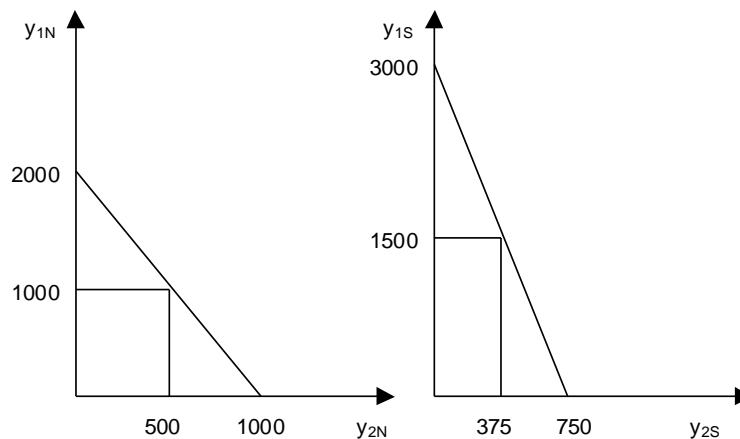
$d_{1N} = 0,5y_N = y_{1N}$ and $d_{2N} = 0,5y_N/2 = y_{2N}$. This implies: $4000 = y_N + y_N = 2y_N$, hence: $y_N = 2000$, $y_{1N} = 1000$, $y_{2N} = 500$.

- **South**

PPF: $9000 = 3 y_{1S} + 12 y_{2S}$, or else $y_{1S} = 3000 - 4 y_{2S}$.

$p_S = p_{2S}/p_{1S} = TMT = 4$.

$d_{1S} = 0,5y_S = y_{1S}$ and $d_{2S} = 0,5y_S/4 = y_{2S}$. $9000 = 1,5y_N + 1,5y_N = 3y_N$, hence $y_N = 3000$, $y_{1N} = 1500$, $y_{2N} = 375$.



Question #2

North: comparative advantage for good 2 (lower MTR). South: good 1.

Both countries specialize fully since returns to scale are constant.

North: $y_{1N} = 0$; $y_{2N} = 4000/4 = 1000$; $y_N = py_{2N} = 1000p$; $d_{1N} = 500p$; $d_{2N} = 500$.

South: $y_{1S} = 9000/3 = 3000$; $y_{2S} = 0$; $y_S = y_{1S} = 3000$; $d_{1S} = 1500$; $d_{2S} = 1500/p$.

Market clearing:

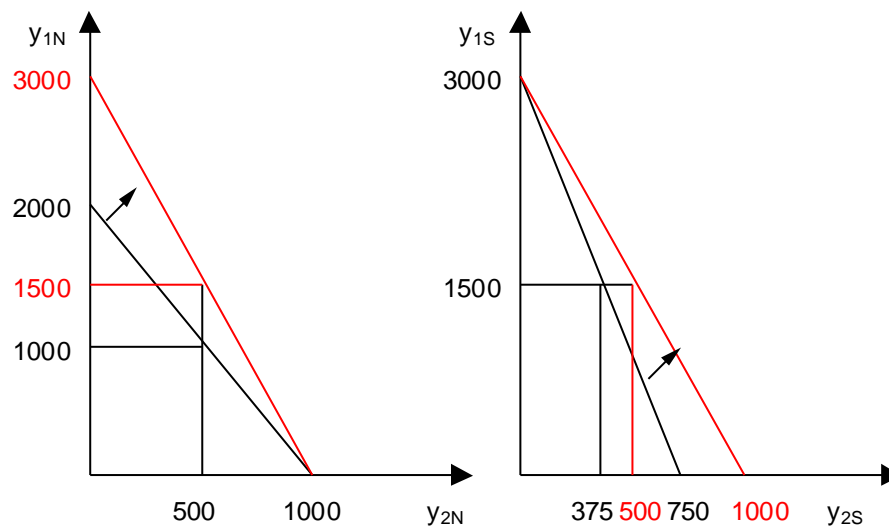
- Good 1 : $3000 = 500p + 1500$, hence $p = 3$.
- One can check that the other market is balanced: $1000 = 500 + 1500/3$

Question #3

North: produces $y_{2N} = 1000$; consumes $d_{1N} = 1500$ and $d_{2N} = 500$, as compared to 1000 and 500 in autarky. Consumption therefore increases. North imports good 1 in quantity $m_{1N} = 1500$ and exports good 2 in quantity $x_{2N} = 500$. Northern trade is balanced: $m_{1N} = p x_{2N}$.

South: produces $y_{1S} = 3000$; consumes $d_{1S} = 1500$ and $d_{2S} = 500$, as compared to 1500 and 375 in autarky. Consumption therefore increases. South exports good 1 and imports good 2.

	North					South				
	Y_{1N}	Y_{2N}	D_{1N}	D_{2N}	X_{2N}	Y_{1S}	Y_{2S}	D_{1S}	D_{2S}	X_{2S}
Autarky	1000	500	1000	500	0	375	1500	375	1500	0
Openness	0	1000	1500	500	500	3000	0	1500	1500	1500



Gains to trade: there is more consumption in each country. It can be seen on the figure that for each utility function strictly concave in (y_{1j}, y_{2j}) , a higher utility level is reached (the equilibrium moves in the North-East direction).