International Economics #3
International trade under imperfect competition

- Limitation of the classics: trade between similar countries, gravity, intra-industry trade
- Theoretical response: imperfect competition
  - "New" theories of international trade
- Limitation of imperfect competition models: extensive versus intensive margins, impact of productivity on trade,…
- Theoretical response: imperfect competition with heterogenous firms
  - "New-new" theories of international trade
- The monopolistic competition model in depth

« Old » and « new » theories of international trade

- ***Ricardo, HOS*** based on perfect competition and constant returns to scale
- Consequences:
  - Market size is irrelevant
  - Explains *inter-industry* trade between countries with different endowments (North-South trade)
  - Does not explain *intra-industry trade* between countries with similar endowments (North-North trade)

- ***New theories of international trade***:
  - Preference for diversity (Dixit-Stiglitz utility functions)

$$U(c) = \left(\sum_{i=1}^{N} c_i^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

  - Product are differentiated
    - Horizontally (varieties)
    - Vertically (qualities)
  - Monopolistic competition with fixed costs.
- References:
  - Robinson (1930), Chamberlin (1936)
  - Lancaster (1980), Helpman (1981)
  - Krugman (1979, 1980) Helpman, Krugman (1985)

➢ ***Explains intra-industry trade***

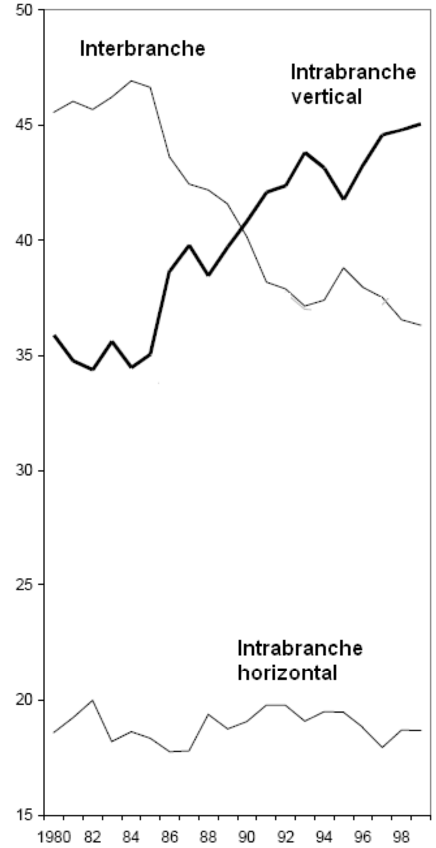
Intra-industry trade

Top-10 country couples for intra-industry trade (% of bilateral trade, 2000)

Top total IIT shares (per cent)		
Germany	France	86.20
Netherlands	Belgium and Luxembourg	85.01
France	Belgium and Luxembourg	80.42
France	United King-dom	77.08
Germany	Switzerland	76.99
Germany	Belgium and Luxembourg	76.83
Austria	Germany	76.63
France	Spain	76.55
Germany	Netherlands	76.01
Canada	United States	73.55

Source: Fontagné, Freudenberg &Gaulier (2006)

Intra-EU trade, 1980-1999



The Krugman (1980) model

<u>Ingredients</u>

- **Monopolistic competition**
- **Iso-elastic preferences**
- **Increasing returns to scale (fixed cost)**
- **Proportional transportation costs**

Consumption (Dixit-Stiglitz)

- L identical individuals who work, consume and hold the firms. Equivalent to one representative individual supplying L units of labor.

- The representative agent consumes a ***continuum of differentiated goods*** $\omega \in \Omega$:

$$\max_{q(\omega)} U = \left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

$\sigma > 1$ is the elasticity of substitution across varieties

- Under the budget constraint: $\int_{\Omega} p(\omega) q(\omega) d\omega = wL$ (no capital, no profit)

- This yields the demand function:

$$q(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\sigma} \frac{wL}{P}$$

with $P = \left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$

Note that the elasticity of substitution is constant whatever the number of varieties.

Interpretation of P

$$P = \left(\int_\Omega p(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}$$

- P is the consumer price index : by definition, ***the consumer price index is the aggregate of p(ω) such that the utility of the real income (i.e. of the nominal income divided by the price index) is the same whatever the general level of prices***.

- With P calculated along the above formula, the utility of wL/P is independent from the general level of prices. To see this property, just calculate U based on optimal $q(\omega)$:

$$U = \left(\int_\Omega q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} \quad \text{with} \qquad q(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\sigma} \frac{wL}{P}$$

- We then have:

$$U = \left(\int_\Omega \left(\frac{p(\omega)}{P}\right)^{-\sigma\frac{\sigma-1}{\sigma}} \left(\frac{wL}{P}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}} = \frac{wL}{P} P^{\sigma} \left(\int_\Omega \left(p(\omega)\right)^{1-\sigma} d\omega\right)^{\frac{\sigma}{\sigma-1}} = \frac{wL}{P} P^{\sigma} P^{-\sigma} = \frac{wL}{P}$$

- Aggregate utility U is equal to the real income: if both the nominal income wL and the price index P increase by x%, utility stays unchanged.

Interpretation of P

(continued)

$$P = \left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$$ and, at the consumer's optimum, $$U = \frac{wL}{P}$$

- Since σ>1, the price index P is lower than the simple average of prices p(ω).

$$P = \left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} < \int_{\Omega} p(\omega) d\omega$$

- This comes from consumer's preference for diversity: higher diversity increases utility, for given prices p(ω); it is as if real income would grow thanks to a lower price index P.
- For a given nominal income wL, the price index P varies inversely to utility.
- ***In the Krugman (1980) model, international trade raises utility through a rise in the diversity of products available to the consumer (not through efficiency gains as in the Ricardian and HOS models of trade).***

Production

- Each firm produces one variety ω for which it has a ***monopole***.
- ***Fixed cost***: to produce q(ω), the firm uses a volume of labor equal to: $l(q) = f + \frac{q}{\varphi}$ (with $f > 0$ and $\varphi > 0$)
- ***Optimal price*** : $p(\omega) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}$ σ/(σ-1) is the ***markup***
- Profit : $\pi(\omega) = p(\omega)q(\omega) - \left(f + \frac{q(\omega)}{\varphi}\right)w = w\left(\frac{q(\omega)}{(\sigma-1)\varphi} - f\right)$
- Free entry: $\pi(\omega) = 0 \Rightarrow$ $q(\omega) = (\sigma - 1)\varphi\, f$
- All firms produce the same quantity at the same price. Hence ω can be omitted in the following.
- ***Number of firms***: *n* such that $n\left(f + \frac{q}{\varphi}\right) = L \Rightarrow$ $n = \frac{L}{f + \frac{(\sigma-1)\varphi f}{\varphi}} = \frac{L}{\sigma f}$

The number of firms depends on the size of the country (*L*), on fixed costs (*f*) and on the elasticity of substitution σ. Higher fixed costs or more competition across varieties (higher σ) will reduce the number of firms in the long run. This is because both σ and *f* reduce the profit of each firm.

Back to the price index

- Price of each variety: $$p(\omega) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}$$
- Price index: $$P = \left(\int_\Omega p(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}$$
- Replace p(ω) by its value: $$P = \left(\int_\Omega \left(\frac{\sigma}{\sigma - 1}\frac{w}{\varphi}\right)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}\left(\int_\Omega d\omega\right)^{\frac{1}{1-\sigma}}$$
- You finally obtain: $$P = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi} n^{\frac{1}{1-\sigma}}$$

  The price index is lower the more varieties you have.
- Replace *n* by its value (cf . number of firms): $$P = \frac{\sigma w}{(\sigma - 1)\varphi}\left(\frac{L}{\sigma f}\right)^{\frac{1}{1-\sigma}}$$
- ***The price index is lower the larger the economy (high L) because this allows more numerous varieties to co-exist. Since utility is inversely related to the consumer price index, welfare is higher in a larger economy (in autarky).***

Two countries

- Assume there are two identical countries except for their sizes: L, L*.

- ***Transportation costs*** are of iceberg type: when 1 unit is shipped by the exporter, the importer only receives 1/τ units, with τ > 1. The rest has melted away.

- Price on the domestic market: $$p(\omega) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}$$

- ***Price for the foreign importer***: $$\tau\, p(\omega) = \frac{\tau\,\sigma}{\sigma - 1}\frac{w}{\varphi}$$

  Note that the price before transportation (FOB price) is the same on both markets because the elasticity of substitution is the same and is constant: the transportation cost is fully passed on the consumer. We'll come back to this point later.

- Total production: $q = q^D + \tau\, q^X$ Total profit: $\pi = pq - \left(f + \frac{q}{\varphi}\right)w = \frac{w}{\sigma\varphi}q - w\,f$

- Free entry: $\pi = 0 \Rightarrow q = (\sigma - 1)\varphi\ f$

- Number of firms: n such that $$n\left(f + \frac{q}{\varphi}\right) = L \Rightarrow n = \frac{L}{\sigma\ f}$$

- ***The number of firms and the production level of each of them is the same as in the autarkic case*** because (1) labor is immobile; (2) the fixed cost and the elasticity of substitution have remained unchanged. (≠ Krugman (1979) where opening up the economy reduces n).

International trade

- Value of aggregate exports: $X = n\tau\, p q^X(\tau\, p)$

  Number of exported varieties → n; Transportation cost → τ; FOB price → p; Foreign demand for each variety → $q^X(\tau\, p)$

- With: $q^X(\tau\, p) = \left(\frac{\tau\, p}{P^*}\right)^{-\sigma} \frac{w^* L^*}{P^*}$
- Replace in X : $X = n\tau\, p\left(\frac{\tau\, p}{P^*}\right)^{-\sigma} \frac{w^* L^*}{P^*} = n\left(\frac{\tau\, p}{P^*}\right)^{1-\sigma} w^* L^*$
- Replace p by $\frac{\sigma}{\sigma-1}\frac{w}{\varphi}$ and n by $\frac{L}{\sigma f}$

$$X = \frac{1}{\sigma f}\left(\frac{\sigma}{(\sigma-1)\varphi}\right)^{1-\sigma} \times L \times L^* \times \left(\frac{\tau w}{P^*}\right)^{1-\sigma} \times w^*$$

International trade
(interpretation)

$$X = \frac{1}{\sigma f} \times L \times L^* \times \left(\frac{\sigma \tau w}{(\sigma - 1)\varphi} \times \frac{1}{P^*} \right)^{1-\sigma} \times w^*$$

- Gravity model:
  - Trade between two countries depends on the product of their sizes (L×L*) and on bilateral transportation costs (often proxied with geographic distance) .

- Intensive and extensive margins:
  - If $\tau \to \infty$, then there is no trade.
  - When τ is no longer infinite, each country starts exporting all its varieties and to import all the other country's varieties: there is a sudden rise in trade through ***extensive margins***.
  - Then, while τ continues to fall, exports of each variety increase but the number of exported varieties stays constant: trade grows through ***intensive margins***.

Welfare gain

- Autarky: $P = pn^{\frac{1}{1-\sigma}}$ and $P^* = p^* n^{*\frac{1}{1-\sigma}}$

- Open economies: $P = \left(np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$

$$P^* = \left(n(\tau p)^{1-\sigma} + n^*(p^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

- With no transportation costs:

$$P = P^* = \left(2np^{1-\sigma}\right)^{\frac{1}{1-\sigma}} = (2n)^{\frac{1}{1-\sigma}} p < (n)^{\frac{1}{1-\sigma}} p \quad \text{since } \sigma > 1$$

Opening up the economy yields a welfare gain deriving from more diversity. In Krugman (1979), there is also a pro-competitive effect (fall in p due to rise in σ).

Wages

Trade balance:

$$\underbrace{\lambda \times L \times L^* \times \left(\frac{\tau w}{P^*}\right)^{1-\sigma} \times w^*}_{X} = \underbrace{\lambda \times L \times L^* \times \left(\frac{\tau w^*}{P}\right)^{1-\sigma} \times w}_{X^*}$$

$$\frac{w}{w^*} = \left(\frac{Lw^{1-\sigma} + L^*(\tau w^*)^{1-\sigma}}{L(\tau w)^{1-\sigma} + L^*(w^*)^{1-\sigma}}\right)^{1/\sigma}$$

- Absent transportation costs ($\tau = 1$), wages are the same in the two countries:

$$\frac{w}{w^*} = \left(\frac{Lw^{1-\sigma} + L^* w^{*1-\sigma}}{Lw^{1-\sigma} + L^* w^{*1-\sigma}}\right)^{\frac{1}{\sigma}} = 1$$

- With high transportation costs ($\tau \to \infty$), wages are higher in the larger country:

$$\frac{w}{w^*} = \left(\frac{Lw^{1-\sigma}}{L^* w^{*1-\sigma}}\right)^{\frac{1}{\sigma}} \Rightarrow \frac{w}{w^*} = \left(\frac{L}{L^*}\right)^{\frac{1}{2\sigma-1}}$$

With transportation costs, wages are higher in the larger country

Interpretation

- Absent transportation costs, the consumers in both countries have access to all varieties in the same conditions. Prices equalize across countries and trade is balanced.
- With a transportation cost, prices are lower in the larger country (say the domestic one, L>L*). In this country, demand for imports is lower (it rises with P). The big country already has access to more varieties than the small one.
- In order for trade to be balanced, exports of the big country should be lowered through a higher marginal cost : w>w*.

$$\lambda \times L \times L^* \times \left(\frac{\tau w}{P^*}\right)^{1-\sigma} \times w^* = \lambda \times L \times L^* \times \left(\frac{\tau w^*}{P}\right)^{1-\sigma} \times w$$

Exports X Imports X*

Consequence: if mobile, the workers should agglomerate in the big country (because w>w*): this is the foundation of ***new economic geography***, which also relies on monopolistic competition (next chapter).

Specialization (Helpman-Krugman)

- ***Two sectors***: one of differentiated goods (cf. previous analysis) and one of homogenous goods. Consumers spend a fixed share μ of their budget in differentiated goods (Cobb-Douglas utility function). Within their spending on differentiated goods, they substitute different varieties with an elasticity of substitution σ (CES utility function).

- The ***homogenous good*** is produced with constant returns to scale (Y=AL) and is traded at no cost. Both countries produce this good. Marginal productivity is normalized to 1. Then, price and wage levels are equal to 1 in this sector for both countries.

- Like in Krugman (1980), all firms producing differentiated goods produce the same quantity q that they sell at the same price p :

$$q = q^D + \tau q^X = \mu \left(\frac{p}{P} \right)^{-\sigma} \frac{wL}{P} + \mu \tau \left(\frac{\tau p}{P^*} \right)^{-\sigma} \frac{w^* L^*}{P^*}$$

- Since $q = q^*$, it can be shown that:

$$n\left(1 - \frac{L}{L^*}\tau^{1-\sigma}\right) = n^*\left(\frac{L}{L^*} - \tau^{1-\sigma}\right)$$

Based on this expression, it is possible to calculate the distribution of differentiated goods production between the two countries as a function of L/L*and τ**.**

Specialization
(continued)

- Start again with: $$n\left(1-\frac{L}{L^*}\tau^{1-\sigma}\right)=n^*\left(\frac{L}{L^*}-\tau^{1-\sigma}\right)$$
- Denoting $\nu = n/n^*$ and $\lambda = L/L^*$: $$\nu=\frac{\lambda-\tau^{1-\sigma}}{1-\lambda\tau^{1-\sigma}}$$
- If $s_n = n/(n+n^*)$, then $s_n = \nu/(1+\nu)$

and finally $$s_n=\frac{\lambda-\tau^{1-\sigma}}{(1+\lambda)\left(1-\tau^{1-\sigma}\right)}$$

Share of differentiated goods production that is located in the home country

Specialization

(interpretation)

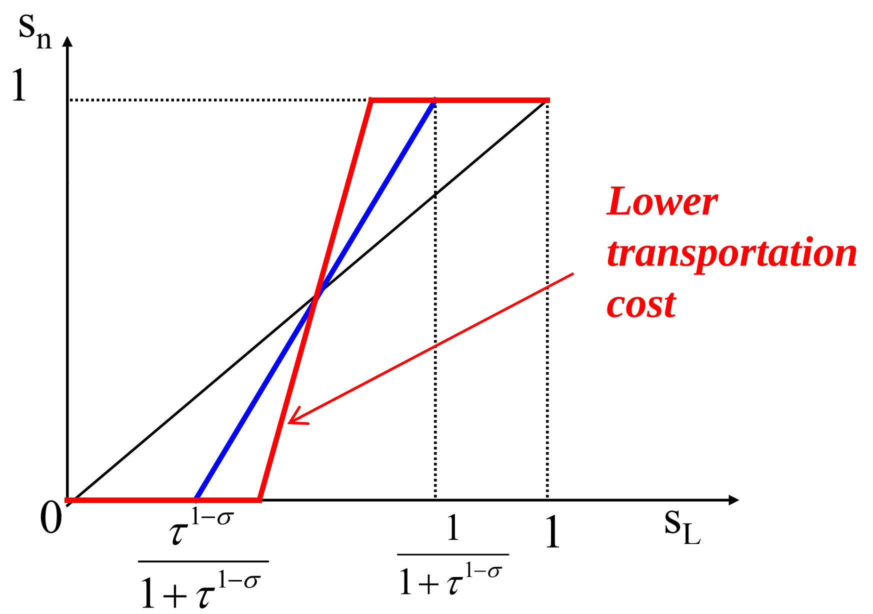
$$s_n = \frac{\lambda - \tau^{1-\sigma}}{(1+\lambda)\left(1-\tau^{1-\sigma}\right)}$$

By definition, $s_n \in [0,1]$.

- If $\lambda < \tau^{1-\sigma}$, $s_n = 0$: if the home country is very small, the production of differentiated goods is entirely located in the foreign economy.
- If $\lambda > 1/\tau^{1-\sigma}$, $s_n = 1$: , the home economy is very large and the production of differentiated goods is entirely located in the home country.
- Between the two thresholds, the larger country hosts a higher proportion of output than its share in the global population. Denoting by s_L the share of the home country in the global population (s_L=L/(L+L*)), the output share s_n writes:

$$s_n = \frac{s_L\left(1+\tau^{1-\sigma}\right) - \tau^{1-\sigma}}{1-\tau^{1-\sigma}}$$

If $\tau \neq \infty$, then the slope of $s_n(s_L)$ is higher than unity: the share in output grows more proportionally than the share in population (***home market effect***).
A ***smaller*** transportation cost ***reinforces*** this effect.

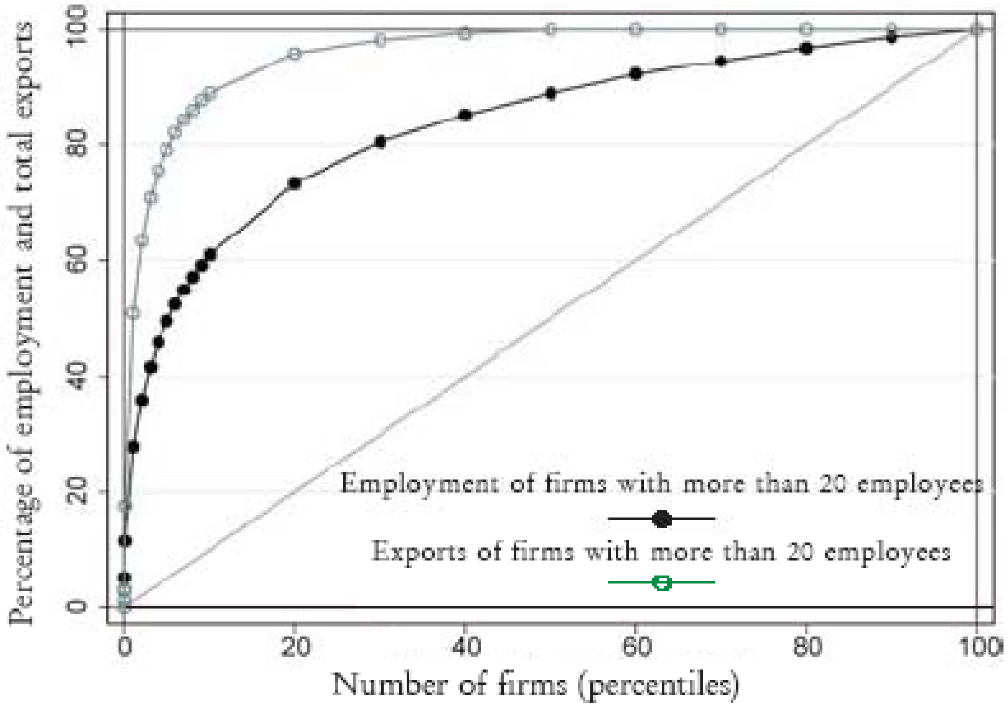


The size range where both countries produce the differentiated good is smaller the smaller transportation costs.

Limitations of the model 1

Exporting firms are larger and more productive than strictly domestic firms: different φ, q

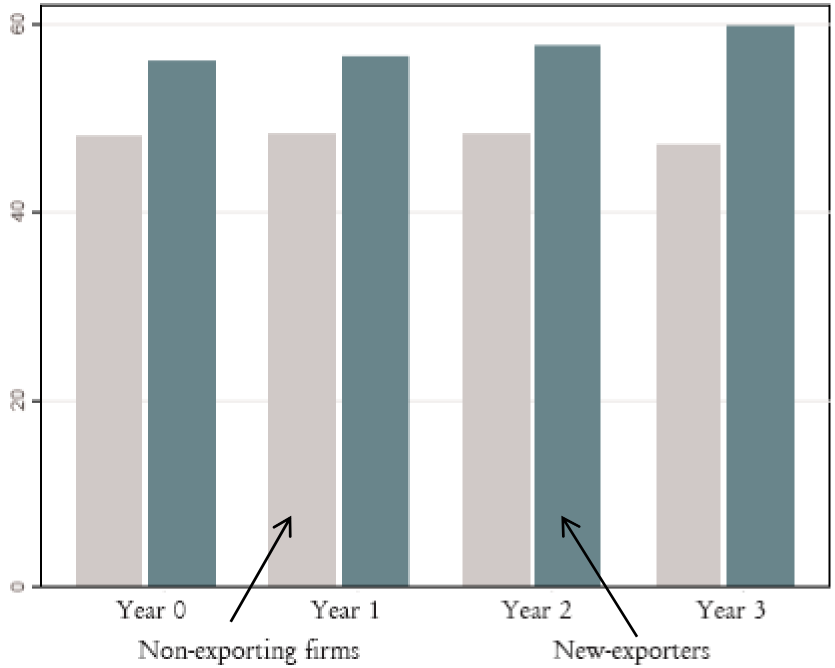
Inequalities between firms, in terms of jobs and exports



Interpretation: amongst the French firms with more than 20 employees, the 20% biggest exporters are responsible for 94% of total exports, but the 20% biggest employers only represent about 75% of total jobs.
Source: French Customs and Excise statistics and Annual Business Survey (INSEE), CEPII calculations.

Source: Crozet & Mayer (2007).

Productivity of firms with more than 20 employees that enter the export markets



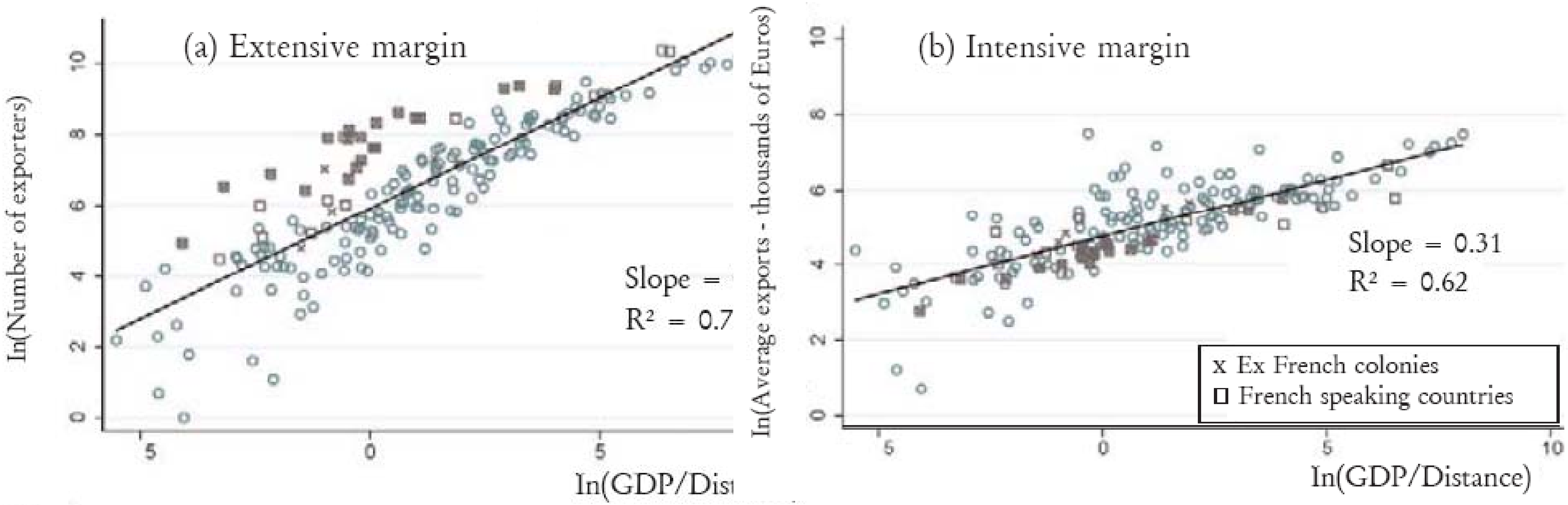
Source : French Customs and Excise statistics and Annual Business Survey (INSEE), CEPII calculations.

Source: Crozet & Mayer (2007).

Limitations of the model 2

Global trade grows through both intensive *and* extensive margins

Intensive and extensive margins of international trade, 2003



Source: Crozet & Mayer (2007).

Distance has more impact through the extensive margin than through the intensive one

The Mélitz model (2003)

<u>Ingredients</u>

- **Monopolistic competition**
- **Isolelastic preferences**
- **Increasing returns to scale**
- **Proportional transportation cost**
- ***Fixed cost of export***
- ***Heterogenous firms in terms of productivity (random)***

From Krugman

- Nominal income R (instead of wL).

$$\max_{q(\omega)} U = \left(\int_\Omega q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \Rightarrow q(\omega) = \left(\frac{p(\omega)}{P} \right)^{-\sigma} \frac{R}{P} \quad \text{with} \quad P = \left(\int_\Omega p(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$$

$$\text{s.c.} \quad \int_\Omega p(\omega) q(\omega) d\omega = R$$

- Production cost $l(q) = f + \frac{q}{\varphi}$ Price: $p(\omega) = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi}$

Model change

- Normalization of wage (=1, cf. existence of an homogenous good with constant returns to scale, like in Helpman-Krugman), ***heterogenous firms in terms of productivity*** φ.

Price: $p(\varphi) = \frac{\sigma}{(\sigma-1)\varphi}$ ***Production*** : $q(\varphi) = \left(\frac{\sigma}{(\sigma-1)\varphi} \right)^{-\sigma} \frac{R}{P^{1-\sigma}}$

Profit : $\pi(\varphi) = p(\varphi) q(\varphi) - \left(f + \frac{q(\varphi)}{\varphi} \right) = \left(\frac{p(\varphi)}{P} \right)^{-\sigma} \frac{R}{P} \left(p(\varphi) - \frac{1}{\varphi} \right) - f = \left(\frac{\sigma-1}{\sigma} \varphi P \right)^{\sigma-1} \frac{R}{\sigma} - f$

Price index

- Productivity φ is randomly drawn for M firms along a distribution $\mu(\varphi)$. The price index writes:

$$P=\left(\int_0^\infty M\mu(\varphi)p(\varphi)^{1-\sigma}d\varphi\right)^{\frac{1}{1-\sigma}}=\left(M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\int_0^\infty \mu(\varphi)\varphi^{\sigma-1}d\varphi\right)^{\frac{1}{1-\sigma}}$$

- Denoting $\tilde{\varphi}=\int_0^\infty \mu(\varphi)\varphi^{\sigma-1}d\varphi$, we have $p(\tilde{\varphi})=\frac{\sigma}{(\sigma-1)\tilde{\varphi}}$ $\Rightarrow$ $P=M^{\frac{1}{1-\sigma}}p(\tilde{\varphi})$

More varieties reduces the price index

Higher average productivity reduces the price index

Exports

- Like in Krugman (1980), there is an iceberg transportation cost $\tau > 1$. Assuming the two countries are identical, the export price, the export volume and the firm's profit are:

$$\tau p(\varphi) = \frac{\tau\sigma}{(\sigma-1)\varphi} \qquad q(\varphi) = \left(\frac{(\sigma-1)\varphi}{\tau\sigma}\right)^{\sigma} P^{\sigma-1}R \qquad \pi(\varphi) = \left(\frac{\sigma-1}{\tau\sigma}\varphi P\right)^{\sigma-1}\frac{R}{\sigma} - f$$

- Export prices can be aggregated:

$$p(\tilde{\varphi}) = \frac{\tau\sigma}{(\sigma-1)\tilde{\varphi}} \qquad \Rightarrow \qquad P = M^{\frac{1}{1-\sigma}}\tau p(\tilde{\varphi})$$

- Three effects of transportation cost: (i) higher import prices; (ii) fewer firms on the domestic market (because less productive foreign firms cannot export); (iii) more low-productivity domestic firms. The price index P falls when the economy opens up and when the transportation cost falls: ***pro-competitive effect***.

- Each firm must decide: (i) whether to produce for the domestic market or not (fixed cost f^D, no transportation cost); (ii) whether to export or not (fixed cost $f^X > f^D$, transportation cost $\tau > 1$).

$$\pi^A(\varphi) = \left(\frac{\sigma-1}{\sigma}\varphi P^A\right)^{\sigma-1}\frac{R}{\sigma} - f^D \qquad \pi^D(\varphi) = \left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1}\frac{R}{\sigma} - f^D \qquad \pi^X(\varphi) = \left(\frac{\sigma-1}{\tau\sigma}\varphi P\right)^{\sigma-1}\frac{R}{\sigma} - f^X$$

(profit in autarky) ***(profit on domestic sales)*** ***(profit on exports)***

Profit

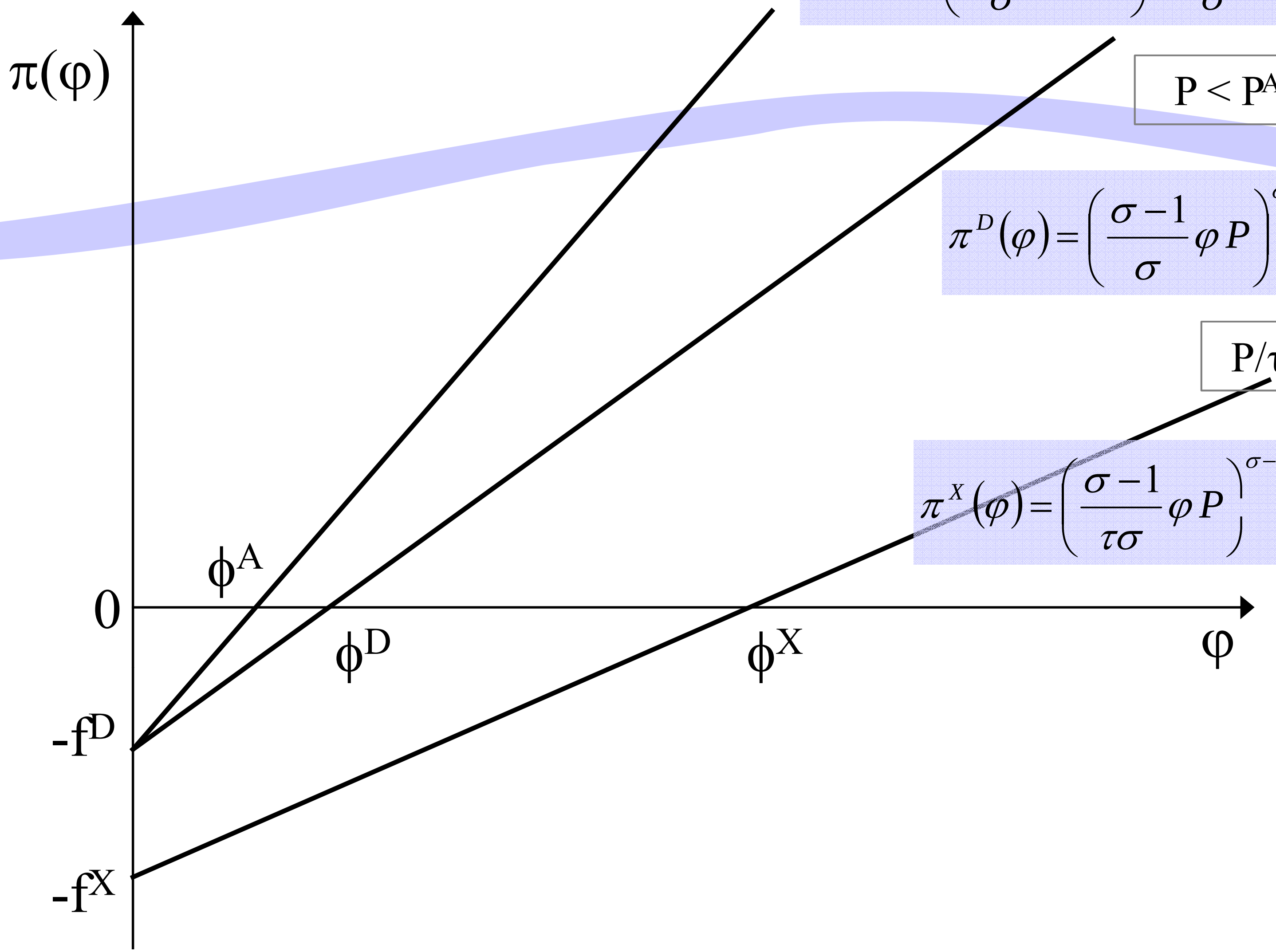
$$\pi^A(\varphi)=\left(\frac{\sigma-1}{\sigma}\varphi P^A\right)^{\sigma-1}\frac{R}{\sigma}-f^D$$

P < P^A

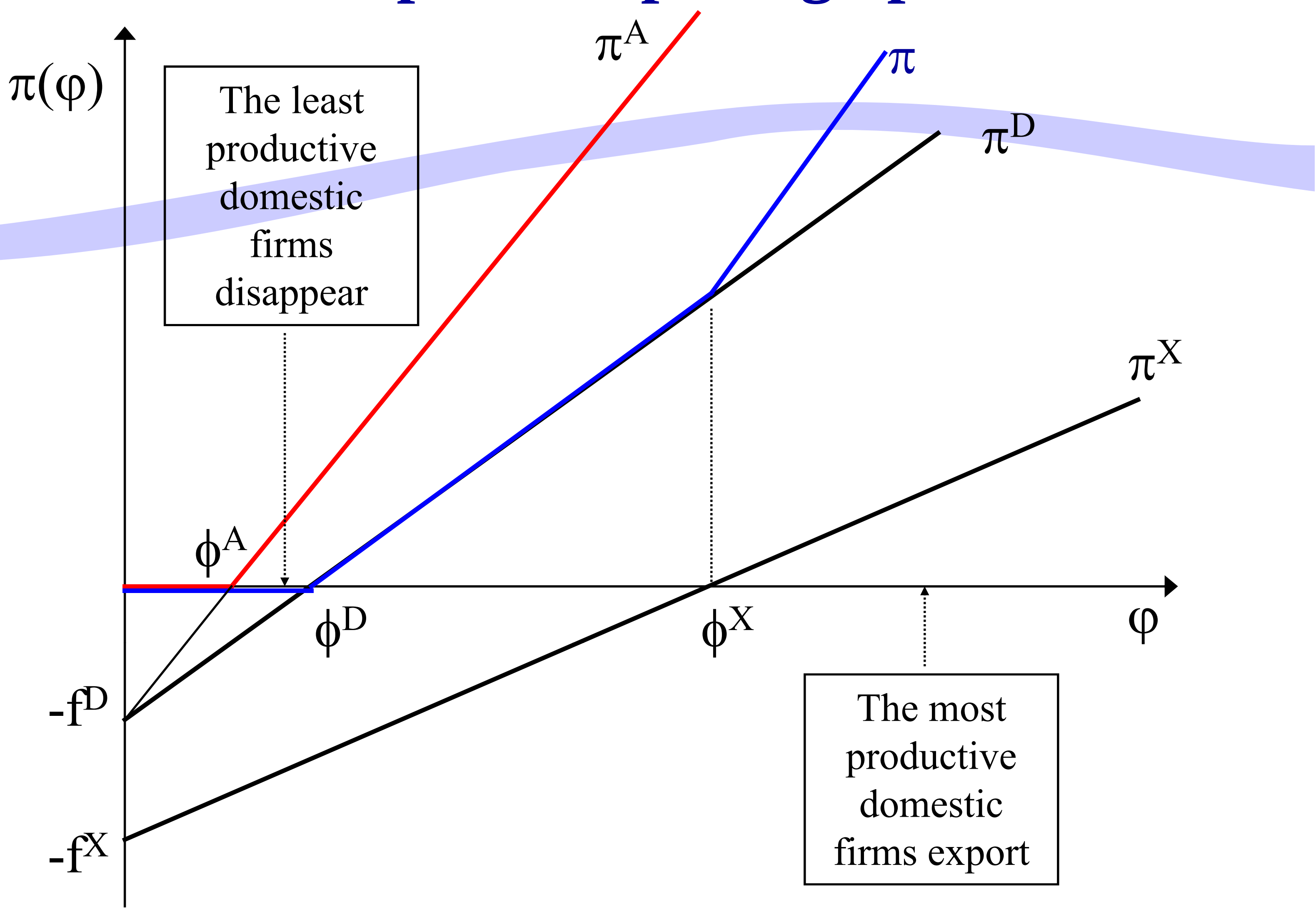
$$\pi^D(\varphi)=\left(\frac{\sigma-1}{\sigma}\varphi P\right)^{\sigma-1}\frac{R}{\sigma}-f^D$$

P/τ < P < P^A

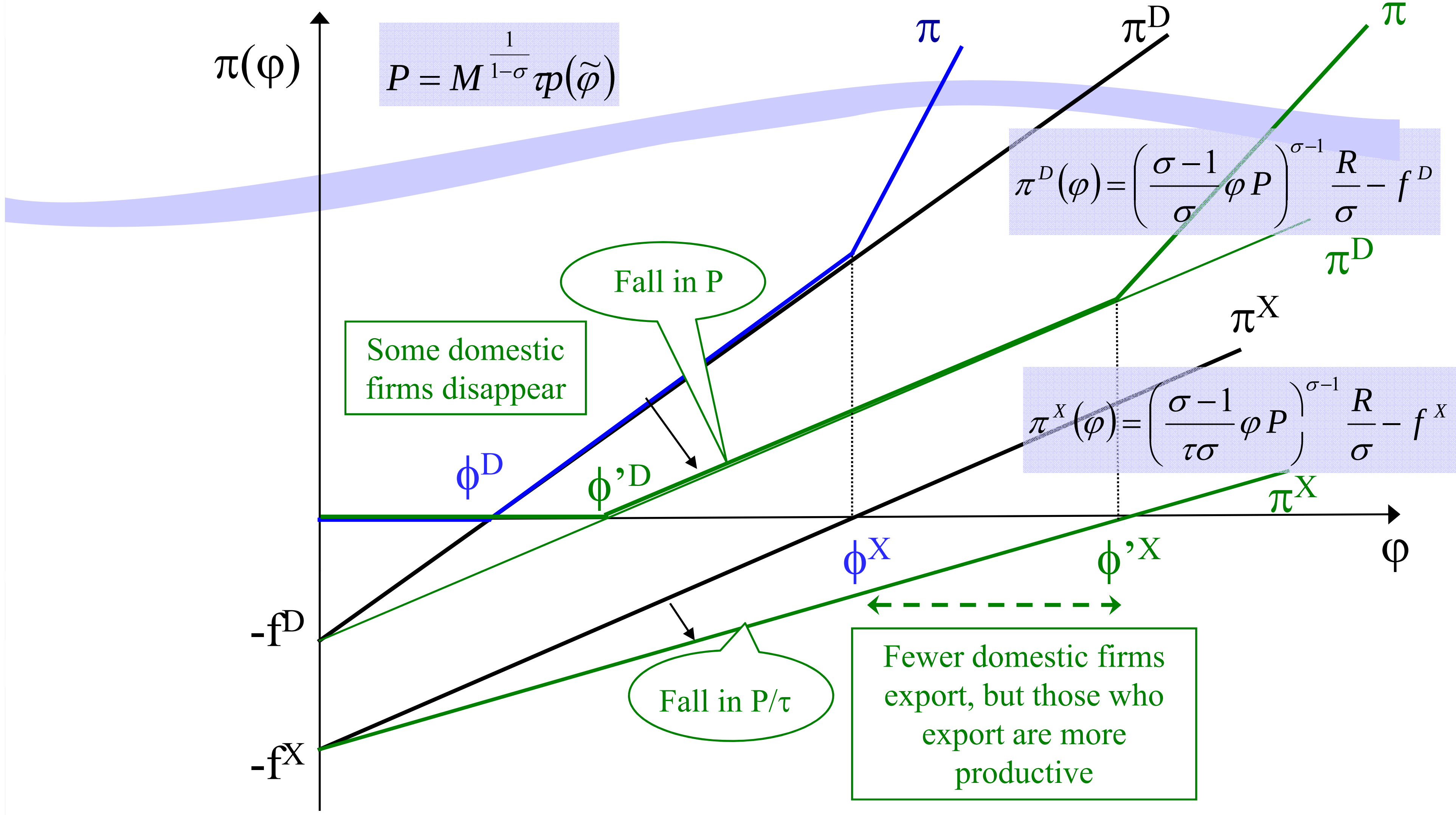
$$\pi^X(\varphi)=\left(\frac{\sigma-1}{\tau\sigma}\varphi P\right)^{\sigma-1}\frac{R}{\sigma}-f^X$$



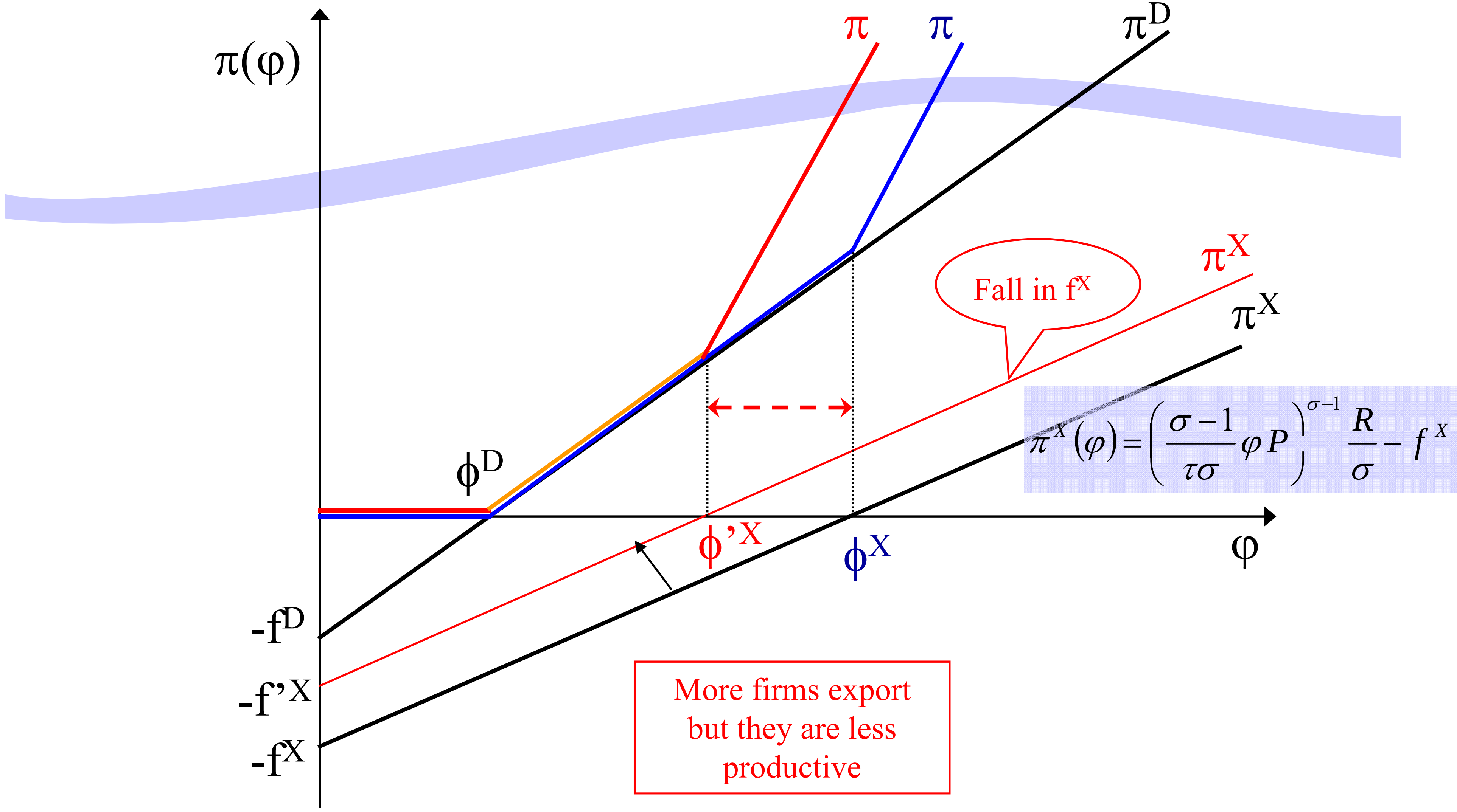
Impact of opening up



Impact of a fall in the transportation cost



Impact of a fall in the fixed exporting cost

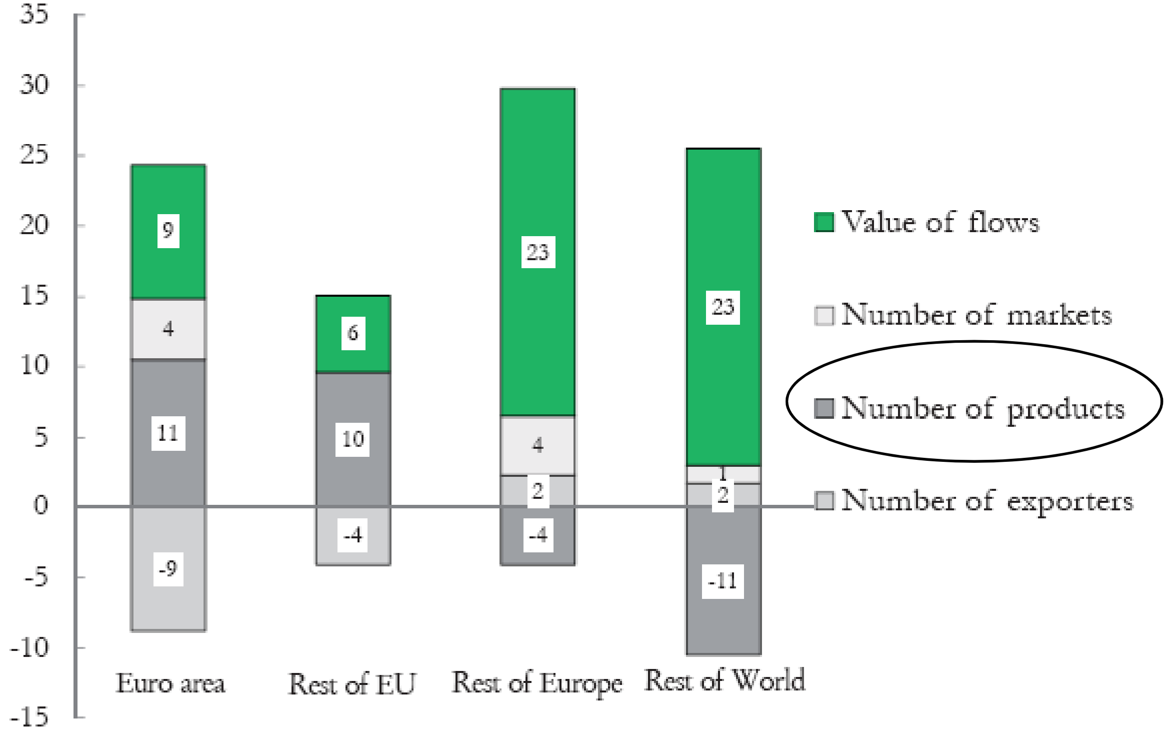


Policy implications

- Trade policies induce reallocations across firms: between industries, but also within industries.
- Although costly, these intra-industry reallocations are not accounted for in standard CGE-based evaluations of trade policies. Such mismatch may contribute to explain resistance to trade liberalization.
- Two trade costs should be distinguished:
  - **Variable costs** (transportation costs, duties): reducing these costs leads to a selection effect (less productive firms stop exporting); trade increases through the ***intensive margin*** (fewer firms export more) ;
  - **Fixed costs**: information, regulations, bureaucracy, red tape: reducing these costs allows more numerous firms, possibly less productive, to export; trade increases through the ***extensive margin***.
- Geographic distance does not only cover transportation costs, but also cultural and regulatory distance, which are fixed costs. This may explain (i) the impact of distance on the extensive margin, and (ii) the persistent impact of distance on trade despite falling transportation costs and tariffs.

A natural experiment: the euro

Graph 1 – Composition of growth in value of French manufacturing exports between 1998 and 2003 according to destination, in %



Econometric analysis

Source: A. Berthou and L. Fontagné (2008-a), *op. cit.*.

The monopolistic competition model in depth

<u>Ingredients</u>

- **Demand function**
- **Optimal price**
- **Price indices in a two-country economy**
- **Wages**
- **Specialization**

How to derive the demand function

In depth

- Lagrangian: $L=\left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right)^{\frac{\sigma}{\sigma-1}}-\mu\left(\int_{\Omega} p(\omega)q(\omega)d\omega-wL\right)$

- Derivate the Lagrangian with respect to q(ω) :

$$\frac{\partial L}{\partial q(\omega)}=q(\omega)^{-\frac{1}{\sigma}}\left(\int_{\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\right)^{\frac{1}{\sigma-1}}-\mu p(\omega)=0$$

- Re-arrange: $q(\omega)^{-\frac{1}{\sigma}}\left(U^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}=\mu p(\omega)\Rightarrow q(\omega)^{-\frac{1}{\sigma}}U^{\frac{1}{\sigma}}=\mu p(\omega)$

- Or, else: $q(\omega)=U\mu^{-\sigma}p(\omega)^{-\sigma}\Rightarrow p(\omega)q(\omega)=U\mu^{-\sigma}p(\omega)^{1-\sigma}$

- Integrate over Ω : $\int_{\Omega} p(\omega)q(\omega)d\omega=U\mu^{-\sigma}\int_{\Omega} p(\omega)^{1-\sigma}d\omega$

- Which also writes: $wL=U\mu^{-\sigma}P^{1-\sigma}$ with $P=\left(\int_{\Omega} p(\omega)^{1-\sigma}d\omega\right)^{\frac{1}{1-\sigma}}$

How to derive the demand function
(continued)

In depth

- Start again with $wL = U\mu^{-\sigma}P^{1-\sigma}$ and $q(\omega)^{-\frac{1}{\sigma}}U^{\frac{1}{\sigma}} = \mu p(\omega)$
- Take μ from the second expression and replace in the first one:

$$\mu = q(\omega)^{-\frac{1}{\sigma}}U^{\frac{1}{\sigma}}p(\omega)^{-1} \Rightarrow wL = Uq(\omega)U^{-1}p(\omega)^{\sigma}P^{1-\sigma}$$

- You get q(ω) as a function of p(ω) :

$$q(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\sigma}\frac{wL}{P}$$

- Everything else being equal, a 1% rise in p(ω) reduces demand q(ω) by σ% (price-elasticity of demand).
- The demand q(ω) also depends on purchasing power wL/P

How to derive the optimal price

In depth

- Write the firm's profit as a function of its price p(ω) :

$$\pi(\omega) = p(\omega)q(\omega) - w\left(f + \frac{q(\omega)}{\varphi}\right) = p(\omega)\left(\frac{p(\omega)}{P}\right)^{-\sigma}\frac{wL}{P} - w\left(f + \frac{1}{\varphi}\left(\frac{p(\omega)}{P}\right)^{-\sigma}\frac{wL}{P}\right)$$

- Maximize the profit (first-order condition) :

$$\frac{\partial \pi(\omega)}{\partial p(\omega)} = \frac{wL}{P}\left[(1-\sigma)\left(\frac{p(\omega)}{P}\right)^{-\sigma} + \frac{\sigma w}{\varphi P}\left(\frac{p(\omega)}{P}\right)^{-\sigma-1}\right] = 0$$

- Then you have: $\sigma - 1 = \frac{\sigma w}{\varphi P}\left(\frac{p(\omega)}{P}\right)^{-1}$

- And finally: $p(\omega) = \frac{\sigma}{\sigma - 1}\frac{w}{\varphi}$

Mark-up — Marginal cost

Price indices in a two-country economy

In depth

- The price index now writes: $P=\left(\int_{\Omega} p(\omega)^{1-\sigma} d\omega+\int_{\Omega^*}(\tau p^*(\omega))^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}$
- At the symmetric equilibrium (p (ω)=p): $P=\left(p^{1-\sigma}\int_{\Omega} d\omega+(\tau p^*)^{1-\sigma}\int_{\Omega^*} d\omega\right)^{\frac{1}{1-\sigma}}$
- And finally: $P=\left(np^{1-\sigma}+n^*(\tau p^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$
- Similarly: $P^*=\left(n(\tau p)^{1-\sigma}+n^*(p^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$
- Absent transportation costs (τ = 1), if marginal costs are the same in the two countries (so that p = p*), the two indices are equal whatever the relative size of the two countries (because both countries have access to the same varieties in the same conditions) :

$$P=P^*=\left(2np^{1-\sigma}\right)^{\frac{1}{1-\sigma}}=(2n)^{\frac{1}{1-\sigma}}p$$

- Both indices are lower than those in autarky, which are:

$$P=pn^{\frac{1}{1-\sigma}} \quad \text{and} \quad P^*=p^*n^{*\frac{1}{1-\sigma}}$$

- ***Opening up the economy thus has a positive impact on welfare,*** for given wages (U=wL/P).

Wages

In depth

- We have expressed prices p(ω) and P as functions of nominal income wL, based on consumer's and firm's optimization.
- L is exogenous but w is endogenous.
- In order to derive the wage level, you need to introduce one last equation: goods market equilibrium.
- Due to the Walras law, it is equivalent to rely on (i) the domestic market; (ii) the foreign market; (iii) the trade balance.
- ***Trade balance***: X = X*

$$\lambda \times L \times L^* \times \left(\frac{\tau w}{P^*}\right)^{1-\sigma} \times w^* = \lambda \times L \times L^* \times \left(\frac{\tau w^*}{P}\right)^{1-\sigma} \times w$$

- Then: $\left(\frac{w}{w^*}\right)^{-\sigma} = \left(\frac{P^*}{P}\right)^{1-\sigma} \Rightarrow \frac{w}{w^*} = \left(\frac{P}{P^*}\right)^{\frac{1-\sigma}{\sigma}}$

- Use: $\left(\frac{P}{P^*}\right)^{1-\sigma} = \frac{np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma}}{n(\tau p)^{1-\sigma} + n^*(p^*)^{1-\sigma}} = \frac{Lw^{1-\sigma} + L^*(\tau w^*)^{1-\sigma}}{L(\tau w)^{1-\sigma} + L^*(w^*)^{1-\sigma}}$

- You get: $\frac{w}{w^*} = \left(\frac{Lw^{1-\sigma} + L^*(\tau w^*)^{1-\sigma}}{L(\tau w)^{1-\sigma} + L^*(w^*)^{1-\sigma}}\right)^{1/\sigma}$

Specialization (Helpman-Krugman)

In depth

- All firms producing differentiated goods produce the same quantity q that they sell at the same price p :

$$q = q^D + \tau q^X = \mu \left(\frac{p}{P} \right)^{-\sigma} \frac{wL}{P} + \mu\tau \left(\frac{\tau p}{P^*} \right)^{-\sigma} \frac{w^* L^*}{P^*}$$

- Replace price indices by their open-economy expressions:

$$q = \mu \frac{p^{-\sigma}}{np^{1-\sigma} + n^*(\tau p^*)^{1-\sigma}} wL + \mu\tau \frac{(\tau p)^{-\sigma}}{n(\tau p)^{1-\sigma} + n^* p^{*1-\sigma}} w^* L^*$$

- Perfect labor mobility across sectors (same wage). Assume φ = σ/(σ -1) so that **p(ω) = 1** with w = 1 (normalization). The production of differentiated good writes, for each variety:

$$q = \mu \left(\frac{1}{n + n^* \tau^{1-\sigma}} L + \frac{\tau^{1-\sigma}}{n\tau^{1-\sigma} + n^*} L^* \right) \quad \text{and} \quad q^* = \mu \left(\frac{\tau^{1-\sigma}}{n + n^* \tau^{1-\sigma}} L + \frac{1}{n\tau^{1-\sigma} + n^*} L^* \right)$$

- Since q = q*, we have: $\frac{1}{n + n^* \tau^{1-\sigma}} L + \frac{\tau^{1-\sigma}}{n\tau^{1-\sigma} + n^*} L^* = \frac{\tau^{1-\sigma}}{n + n^* \tau^{1-\sigma}} L + \frac{1}{n\tau^{1-\sigma} + n^*} L^*$
- Re-arranging: $L \frac{1 - \tau^{1-\sigma}}{n + n^* \tau^{1-\sigma}} = L^* \frac{1 - \tau^{1-\sigma}}{n\tau^{1-\sigma} + n^*}$ or else $n + n^* \tau^{1-\sigma} = \frac{L}{L^*} \left(n\tau^{1-\sigma} + n^* \right)$
- Finally:

$$n \left(1 - \frac{L}{L^*} \tau^{1-\sigma} \right) = n^* \left(\frac{L}{L^*} - \tau^{1-\sigma} \right)$$